# REGULARIZATION BY LOWERING AND RAISING ITS ACCURACY IN THE SOLUTION OF INVERSE PROBLEMS OF HEAT CONDUCTION

Yu. M. Matsevityi

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The question of the a priori restriction of computational accuracy in order to obtain stable solutions of inverse problems by the methods of high-precision regularization is posed.

As is well known, inverse problems in field theory, in particular, inverse problems of heat conduction (IPHC), are improperly posed problems because the cause-effect coupling between the input and output parameters of the object under study breaks down. This means that in solving them all or at least one of the Hadamard conditions for a problem to be properly posed (existence, uniqueness, or stability) may not hold [1], i.e., the regularity of the solutions of these problems may break down. Although in principle the questions of the existance and uniqueness of the solutions obtained must be analyzed to the same degree as the questions of stability, the main factor giving rise to irregularity of the solutions is instability (small errors in the starting data can give rise to large errors in the parameters identified).

To determine the paths for solving improperly posed problems Tikhonov's conditions for a problem to be properly posed [2], which make it possible to reduce an improperly posed problem to a properly posed one, are most important. Since the problem becomes properly posed according to Tikhonov if the set of solutions sought is narrowed, it is sometimes called a conditionally properly posed problem [3], while stability is a conditional problem [4].

One way to achieve a proper formulation according to Tikhonov is to restrict the set of possible solutions to a compact set [5]. The question of choice of which one is solved based on physical considerations in each specific problem. For example, in [6] in solving the problem of identification of a constant heat flux with unilateral heating of a flat plate Alifanov employed the properties of thermal regularity [7], and this substantially simplified the determination of the parameter to be identified. In this case, a priori information about the character of the function sought (its constancy) was employed to restrict the set of admissible solutions. By the way, the properties of the identified functions (for example, differentiability, sign-definiteness, continuity, etc.) are often known beforehand. These properties serve as a basis for separating the class of properly posed problems. Methods employing this approach are conditionally regular with restriction by formulation [8]. These are V. K. Ivanov's [4] and M. M. Lavrent'ev's [3] method of quasisolutions, A. N. Tikhonov's method of fitting the interpretation, the method of quasiinversion [9], and others. A priori information about the dependences sought is used quite effectively in the solution of IPHC by the method of spectral influence functions [10], which enables the solution of multiparameter inverse problems, which thus far could be solved only by methods based on the general theory of regularization developed by A. N. Tikhonov 11 . In these methods the class of admissible functions is not prerestricted to a compact set, but rather the functions sought are required to meet certain requirements (for example, smoothness), which ensure that the solutions obtained are stable against small changes in the starting data.

Aside from the methods examined above, conditionally regular methods with restriction on the algorithm are widely used [8]. They include, in particular, the methods of inversion of the algorithm [6], i.e., methods which assume the possibility of inversion of the solution of the direct problem, and methods of successive intervals [12], integral characteristics [13], series expansions [14], and iterations, including methods of adjustment [15-17], The stability of the solutions obtained in these methods is ensured by imposing restrictive conditions

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on the parameters of the conputational algorithms. Restrictions are most often imposed on the approximation step (step regularization), the degree of the approximating polynomial (power-law regularization), and the number of iterations (iterational regularization). These forms of regularization are modifications of the so-called *natural* regularization [6]. This term, on the one hand, may be regarded as suitable, since in order to obtain solutions there is no need to change *artificially* the formulation of the problem or to restrict beforehand the set of solutions sought. Regularity here is determined by the *natural* features of the physical process (for example, the effect of regularization of the thermal regime, associated with the slowness of the propagation of heat) and "viscous" (damping) properties of computational algorithms.

On the other hand, all these methods of regularization actually restrict the *accuracy* of the solutions obtained (by reducing the accuracy of either the mathematical model or the computational process). From this viewpoint they can all be referred to "*accuracy* regularizaton" or "regularization by *reducing the accuracy*."

The restriction of accuracy, achieved in the solution of inverse problems on analog devices, is indeed natural, since it is brought about by the comparatively low accuracy of analog computers (thus, the drawbacks of the latter are replaced by their advantages in solving improperly posed problems — actually experience in the solution of IPHC on analog computers indicates that the solutions obtained are stable).

The question of accuracy regularization, of course, is mentioned here not so much on a terminological level, but rather as a rationalization for computational processes employing natural (or accuracy) regularization methods. Since in these methods ensuring stability of the solutions reduces to restricting the accuracy of the computational process, there must be a relationship between the dimensions of the zone of possible instability (we use this term to refer to the region encompassing the exact solution, within which the approximate solution of the inverse problem is unstable) and the error in the starting data. This relationship, found for the object under study taking into account the method of solution employed and the arrangement of the points of observation, should indicate even before the start of the numerical experiment the accuracy to which the calculations should be taken, without the risk of entering the zone of possible instability mentioned above.

In this paper we have posed the question of the existence of the above-indicated relationship between the restriction on the computational accuracy and the errors in the starting information, so that here we do not talk about ways to establish such a relationship, which, apparently, will have their own peculiarities, specific to the method employed and the object studied. The same concerns also the methods for increasing the accuracy, characteristic for one or another method.

In particular, returning to the method of spectral influence functions mentioned above [10], we note that the combined use of this method with the regionally structural approach to the solution of heat problems is an example of a successful combination of high-precision indicators with good stability of the solutions obtained. This is achieved, on the one hand, by obtaining a more correct approximation for the limiting actions sought (the influence function contains exact information about the geometry of the object), which together with the analytic determination of the main components of the spectral influence functions increases the accuracy of identification. On the other hand, the partitioning of the object of study into regions and the use of regional spectral influence functions have a regularizing effect on the solutions obtained [10], since the boundary action function within the region can be approximated quite coarsely (for example, by a polynomial of degree one to three). At the same time, the indicated approximation with respect to the surface of the entire object is very accurate and, therefore, has virtually no effect on the accuracy indicators of the parameters identified. In particular, in using this method to solve multiparametric IPHC of determining the surface thermal effects in a prismatic body with a rectangular cross section indications of instability appeared only when the accuracy of the calculations was raised up to four decimal places (an error of less than 0.01%).

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## REVIEWS

# THERMAL AND PHYSICAL PROPERTIES OF MAGNETIC FLUIDS

V. E. Fertman UDC 541.18:532.13+532.61+536.2+536.4+536.6

The basic physical properties of dilute magnetic colloidal solutions (a new class of fluids, in which the interaction of the dispersed phase with an external magnetic field is a source of an additional inertial force) were considered in the review [1]. As a result of this interaction, it appears to be possible to enhance the transport of heat in a magnetic fluid by means of a stationary nonuniform magnetic field, which induces thermomagnetic convection in a nonisothermal fluid. In the time since the appearance of this review, extensive experimental data has become available on the thermal and physical properties of concentrated magnetic fluids (volume concentration of the dispersed phase in the interval $0.1 \leq \varphi \leq 0.20$). In addition, new mechanisms enhancing heat transport in concentrated magnetic fluids have been identified. One of them is connected with the microscopic mixing of a fluid with rotation induced by means of solid colloidal particles and aggregates [2]. Rotation can also cause macroscopic motion, which affects the transport of heat in a nonisothermal fluid [3].

It is of interest to summarize the experimental results obtained in the last few years on the thermal and physical properties of magnetic fluids for a wide interval of concentrations of the dispersed phase.

The thermal regime of the most common device using magnetic fluids (magnetic-fluid seals) depends mainly on viscous dissipation inside the working gap [4]. Therefore, we also consider in the present review internal friction in magnetic fluids for strong shear deformations in the presence of a magnetic field.

The technology for obtaining magnetic fluids whose dispersed phase is single-domain magnetite ($Fe_3O_4$) particles is the most developed. The mass per unit volume of fluid is composed of the masses of three components:

$$\rho_{mf} = \rho_0\varphi_0 + \rho_M\varphi + \rho_{SAM}[1 - (\varphi_0 + \varphi)]. \tag{1}$$

A. V. Lykov Institute of Heat and Mass Transfer, Academy of Sciences of the Belorussian SSR, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 53, No. 3, pp. 502-512, September, 1987. Original article submitted September 16, 1986.

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